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Toward a Probabilistic Preliminary Design Criterion for Buckling Critical Composite Shells

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A probability-based analysis method for predicting buckling loads of compression-loaded laminated composite shells is presented, and its potential as a basis for a new shell-stability design criterion is demonstrated and discussed. In particular, a database containing information about specimen geometry, material properties, and measured initial geometric imperfections for a selected group of laminated-composite cylindrical shells is used to calculate new buckling-load "knockdown factors." These knockdown factors are shown to be substantially improved and hence much less conservative than the corresponding deterministic knockdown factors that are presently used by industry. The probability integral associated with the analysis is evaluated by using two methods; that is, by using the exact Monte Carlo method and by using an approximate first-order second-moment method. A comparison of the results from these two methods indicates that the first-order second-moment method yields results that are conservative for the shells considered. Furthermore, the results show that the improved, reliability-based knockdown factor presented always yields a safe estimate of the buckling load for the shells examined.

Nomenclature

A_{io} = Fourier coefficients of the measured initial imperfections [see Eq. (7)]
a = $E(Z)$, mean value of Z
$C_{k\ell}$, $D_{k\ell}$ = Fourier coefficients of the measured initial imperfections [see Eq. (7)]
c = $\sqrt{[3(1-\nu_{12}^2)]}$
$\mathrm{cov}(X_j, X_k)$ = covariance of the random variables X_j and X_k [see Eq. (11)]
$E(X_j)$ = mean value of the random variable X_j [see Eq. (10)]
E_{11}, E_{22} = Young's moduli in the 1 and 2 directions
$\mathrm{erf}(\beta)$ = error function
$f_{\bar{X}}(\bar{x})$ = joint probability density function
$f_Z(t)$ = probability density function
G_{12} = shear modulus in the 1-2 plane
$g(\bar{X})$ = response (or limit-state) function
i, k, ℓ = integers
L = shell length
$N_{c\ell}$ = $E_{11}t^2/cR$, classical buckling load of the corresponding isotropic shell
P = axial compression load
P_c = $2\pi R\lambda_c^m N_{c\ell}$, critical (lowest) buckling load using membrane prebuckling
P_{exp} = experimental buckling load
$P_f(\lambda)$ = probability of failure
R = shell radius
$R(\lambda)$ = reliability function
t = shell wall thickness
$\mathrm{var}(Z)$ = variance of Z
$\bar{W}$ = out-of-plane imperfection
$\bar{X}$ = vector representing the random variables of the problem
$Z(\bar{X})$ = linearized response (or limit-state) function
β = reliability index [see Eq. (21)]
γ = traditional empirical knockdown factor
Δ_{asy} = asymmetric component of the rms value of the measured initial imperfection [see Eq. (9)]
Δ_{axi} = axisymmetric component of the rms value of the measured initial imperfection [see Eq. (9)]
Δ_{rms} = rms value of the measured initial imperfection [see Eq. (8)]
Λ_s = normalized random collapse load
λ = normalized loading parameter ($= P/P_c$)
λ_a = reliability-based improved knockdown factor (see Fig. 2)
λ_c^m = 0.398, critical (lowest) normalized buckling load of shell AWCYL-11-1 using membrane prebucking
λ_{c_i} = eigenvalue corresponding to the axisymmetric mode with i half-waves in the axial direction [see Eq. (7)]
$\lambda_{c,k\ell}$ = eigenvalue corresponding to the asymmetric mode with k half-waves in the axial direction and ℓ full waves in the circumferential direction [see Eq. (7)]
ν_{12} = Poisson's ratio
$\bar{\xi}_1$ = $-\Delta_{axi}$, equivalent axisymmetric imperfection amplitude
$\bar{\xi}_2$ = Δ_{asy}, equivalent asymmetric imperfection amplitude
ρ_a = λ_a/λ_c^m, reliability-based improved renormalized knockdown factor (see Fig. 2)
ρ_{c_i} = renormalized axisymmetric eigenvalue, $\lambda_{c_i}/0.398$
$\rho_{c,k\ell}$ = renormalized asymmetric eigenvalue, $\lambda_{c,k\ell}/0.398$
σ_Z = $\sqrt{[\mathrm{var}(Z)]}$, standard deviation of Z
$\phi(\beta)$ = standard normal probability distribution

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Introduction

THE difficulty in predicting the behavior of compression-loaded thin-walled cylindrical shells continues to concern design engineers. Thanks to extensive research programs conducted in the late 1960s and early 1970s and the contributions of many eminent scientists, it is now known that buckling strength can be affected significantly by the uncertainties in the definition of loads, material properties, geometric variables, edge support conditions, engineering models, and the accuracy of the analysis tools used in the design phase.

Presently, the NASA design criteria monographs, e.g., NASA SP-8007,[1] from the late 1960s are used, to a wide extent, in industry to design buckling-critical shells. The data in these monographs account for design uncertainties by using a "lump-sum" safety factor that is determined from experimental data. This safety factor is often referred to by designers as the "empirical knockdown factor γ." Structural design verification is achieved by applying the worst-case loading to the structure and testing to failure. However, it has been found that using this approach to design buckling-critical shells often leads to overly conservative designs. The reason for this deficiency is primarily because the empirical knockdown factor is based on a lower-bound design curve derived from a large amount of experimental data for which the test pedigree and other things like initial geometric imperfections and boundary conditions are not well defined. In addition, these knockdown factors do not include information for shell structures made from advanced composite materials. Thus, new, improved knockdown factors are expected to yield significant weight savings and improved performance in the design of buckling-critical shells. Further, the new knockdown factors presented herein will, for the first time, provide design information for compression-loaded composite laminated shells. One approach that appears to have potential for realizing these improvements is a reliability-based approach.

Following the pioneering work of Freudenthal[2] on structural safety and reliability, around 1970, the use of probabilistic methods to assess the safety of structures was proposed.[3,4] Recent papers, for example, Refs. 5–7, have proposed a new reliability-based design procedure for buckling-critical isotropic shells. However, similar results for compression-loaded laminated composite shells do not exist in the literature.

The focus of the present paper is on the extension of the probabilistic approach to buckling of compression-loaded laminated composite shells. In particular, the goal is to formulate a new and improved scientific knockdown factor that is based on statistical information obtained from measurements made on families of composite shells. As a minimum, the measurements include initial geometric imperfections, but can also include other critical details such as shell-wall thickness variations, material property variations, nonuniform load introduction, and boundary condition effects. With this approach, it is believed that a scientific, reliability-based knockdown factor will result in a better engineered, better designed, and safer structure by quantifying, and supplying understanding of, problem uncertainties such as initial imperfections and their interaction with the design variables.

Thus, the objective of the present paper is to demonstrate a reliability-based buckling-analysis approach that can be used as the basis of a new preliminary design criterion for buckling-critical shells, particularly for shells made of advanced laminated-composite materials. To accomplish this objective, a probability-based analysis method for buckling of compression-loaded cylindrical shells is presented first. In this method, a database that contains information about test specimen geometry, material properties, and measured initial imperfections for a selected group of laminated-composite cylindrical shells is used to calculate reliability-based buckling-load knockdown factors. In addition, the probability integral associated with the analysis is evaluated by using an approximate, less computationally intensive, first-order second-moment (FOSM) method, and these results are compared to corresponding results obtained by using the exact, computationally intensive Monte Carlo method. Then, the results of the reliability-based analysis are compared with results based on NASA design guidelines for buckling of compression-loaded isotropic cylindrical shells.

Reliability-Based Buckling Analysis

The buckling problem for axially compressed cylinders can best be formulated in terms of a response (or limit state) function such as

$$g(\bar{X}) = \Lambda_s(\bar{X}) - \lambda \tag{1}$$

The components of the random vector X_i can be Fourier coefficients of the initial geometric imperfections and other parameters that quantify the uncertainties in the specified boundary conditions, the constitutive equation used to describe the nonlinear material behavior, the shell-wall thickness distribution, residual stresses, etc. Notice that the evaluation of the response function generally involves the solution of a complicated nonlinear structural analysis problem that typically requires a detailed, and possibly large, finite element model. However, with today's computational resources, a complex nonlinear analysis in itself does not pose any insurmountable difficulties. The response function $g(\bar{X}) = 0$ separates the variable space into a "safe region," where $g(\bar{X}) > 0$, and a "failure region," where $g(\bar{X}) \leq 0$. The reliability $R(\lambda)$ is calculated from the probability of failure $P_f(\lambda)$ by

$$R(\lambda) = 1 - P_f(\lambda) \tag{2}$$

where

$$P_f(\lambda) = \text{Prob}\{g(\bar{X}) \leq 0\} = \int \cdots \int_{g(\bar{X}) \leq 0} f_{\bar{X}}(\bar{x})\, \mathrm{d}\bar{x} \tag{3}$$

and $f_{\bar{X}}(\bar{x})$ is the joint probability density function of the random variables involved. The credibility of this approach depends on two factors: the accuracy of the mechanical model used to calculate the limit-state function and the accuracy of the probabilistic techniques used to evaluate the multidimensional probability integral.

The FOSM method is an approximate method for evaluating multidimensional probability integrals that is not as computationally demanding as exact methods such as the Monte Carlo method. The FOSM method involves linearizing the response function such that $g(\bar{X}) \approx Z(\bar{X})$ at the mean point and requires knowledge of the distribution of the random vector $\bar{X}$. By assuming that both $Z(\bar{X})$ and $\bar{X}$ are normally distributed, then

$$\begin{aligned} R(\lambda) &= 1 - \text{Prob}(Z \leq 0) \\ &= 1 - \int_{-\infty}^{0} f_Z(t)\, \mathrm{d}t \end{aligned} \tag{4}$$

where the probability density function $f_Z(t)$ is given by

$$f_Z(t) = \left(1\big/\sigma_Z\sqrt{2\pi}\right) \exp\left\{-\tfrac{1}{2}[(t-a)/\sigma_Z]^2\right\} \tag{5}$$

and $a = E(Z)$, $\sigma_Z = \sqrt{[var(Z)]}$. It has been shown in earlier works (e.g., Refs. 5 and 6) that with this approach the solution of the reliability-based buckling problem defined by Eq. (3) can be reduced to a series of $n+1$ deterministic buckling-load calculations, where n is the number of random variables used.

Numerical Results

To include the effects of initial geometric imperfections in the analytical model in a simple manner, for the purpose of demonstrating the reliability-based approach the following two-mode initial imperfection model that consists of an axisymmetric and an asymmetric component was employed:

$$\bar{W}/t = \bar{\xi}_1 \cos i\pi(x/L) + \bar{\xi}_2 \sin k\pi(x/L) \cos \ell(y/R) \tag{6}$$

From the work of Koiter[8] and Budiansky and Hutchinson,[9] it is known that the worst imperfection shapes (the ones that give the biggest decrease in the buckling load for a given amplitude) are those that are similar to the critical bifurcation buckling mode shapes. Thus, predicted bifurcation buckling mode shapes are used in the present analysis to form a worst-case imperfection shape. To apply the first-order second-moment method, the amplitudes of the axisymmetric and asymmetric components in the imperfection model given in Eq. (6) are set equal to the rms values of the corresponding measured initial imperfections. The initial imperfection is modeled using the following alternate double Fourier representation:

$$\bar{W}(x, y) = t \sum_{i=1}^{n_1} A_{0i} \cos i\pi \frac{x}{L} + t \sum_{k=1}^{n_1} \sum_{\ell=2}^{n_2} \sin k\pi \frac{x}{L} \left(C_{\mathrm{kl}} \cos \ell \frac{x}{L} + D_{\mathrm{kl}} \sin \ell \frac{y}{R} \right) \tag{7}$$

Then, by definition, the rms value of the measured initial imperfection is

$$\Delta_{\mathrm{rms}}^2 = \frac{1}{2\pi RL} \int_0^{2\pi R} \int_0^L [\bar{W}(x, y)]^2 \,\mathrm{d}x\,\mathrm{d}y = \frac{t^2}{4} \left[2 \sum_{i=1}^{n_1} A_{0i}^2 + \sum_{k=1}^{n_1} \sum_{\ell=3}^{n_2} \left(C_{\mathrm{kl}}^2 + D_{\mathrm{kl}}^2 \right) \right] \tag{8}$$

Thus,

$$\left(\frac{\Delta_{\mathrm{rms}}}{t} \right)^2 = \frac{1}{2} \sum_{i=1}^{n_1} A_{0i}^2 + \frac{1}{4} \sum_{k=1}^{n_1} \sum_{\ell=3}^{n_2} \left(C_{\mathrm{kl}}^2 + D_{\mathrm{kl}}^2 \right) = \Delta_{\mathrm{axi}}^2 + \Delta_{\mathrm{asy}}^2 \tag{9}$$

The measured initial shape of one of the composite shells used (AWCYL-11-1) is shown in Fig. 1. As can be seen, the initial imperfections are dominated by the $\ell = 2$ mode, which represents an ovalization of the cylinder. However, the eigenvalue from a linear bifurcation analysis corresponding to the asymmetric mode ($k = 1$, $\ell = 2$) is equal to 4.106, which is about 10 times bigger than the lowest eigenvalue $\lambda_c = 0.398$. Based on this fact, it was decided to neglect the contribution of the ovalization (the $\ell = 2$) mode when calculating the rms values of the measured initial imperfections. Otherwise, the resulting numerical model used for the buckling load calculations will predict unrealistically low buckling loads because the amplitude of the imperfection would be artificially inflated as a result of the large magnitude of the ovalization (the $\ell = 2$) mode.

In this paper, the values $\bar{\xi}_1 = -\Delta_{\mathrm{axi}}$ and $\bar{\xi}_2 = \Delta_{\mathrm{asy}}$ denote the equivalent axisymmetric and asymmetric imperfection amplitudes, respectively. A listing of the equivalent imperfection amplitudes obtained for nine composite laminated shells fabricated and tested at NASA Langley Research Center is given in Table 1. The buckling load calculations were carried out with the code TWOMOD.[10] This code is based on a Galerkin-type solution of the nonlinear Donnell-type equations for imperfect anisotropic cylinders. The theoretical buckling load is defined as the value of the nondimensional loading parameter $\lambda = P/P_{\mathrm{cl}}$ at the limit point of the prebuckling state. For the statistical calculations, the data associated with laminated composite shells are used. A listing of the geometric and material properties of the nine NASA shells tested is given in Table 2. Typical results of the data reduction for the measured initial imperfections of these shells are tabulated in Refs. 11 and 12. The mean values and the variance-covariance matrices of the random variables X_i were evaluated by using the following ensemble averages for a sample of the experimentally measured initial imperfections:

$$\bar{\xi}_j = E(X_j) = \frac{1}{M} \sum_{m=1}^{M} X_j^{(m)} \tag{10}$$

$$\mathrm{cov}(X_j, X_k) = \frac{1}{M-1} \sum_{m=1}^{M} \left[X_j^{(m)} - \bar{\xi}_j \right] \left[X_k^{(m)} - \bar{\xi}_k \right] \tag{11}$$

yielding

$$E(X_1) = -0.00334$$

$$E(X_2) = -0.00498 \tag{12}$$

$$\mathrm{cov}(X_1, X_2) = \begin{bmatrix} 0.0000108 & -0.000101 \\ -0.000101 & 0.00121 \end{bmatrix} \tag{13}$$

Table 1 Values of rms imperfection amplitudes and the sample mean vector

Shell	$\bar{\xi}_1 = -\Delta_{\mathrm{axi}}$	$\bar{\xi}_2 = -\Delta_{\mathrm{asy}}$
AWCYL-1-1	$-0.27460 \cdot 10^{-3}$	$0.15395 \cdot 10^{-1}$
AWCYL-2-1	$-0.33547 \cdot 10^{-2}$	$0.68915 \cdot 10^{-1}$
AWCYL-3-1	$-0.23708 \cdot 10^{-2}$	$0.30062 \cdot 10^{-1}$
AWCYL-4-1	$-0.40151 \cdot 10^{-2}$	$0.37061 \cdot 10^{-1}$
AWCYL-5-1	$-0.28099 \cdot 10^{-2}$	$0.32179 \cdot 10^{-1}$
AWCYL-11-1	$-0.22241 \cdot 10^{-2}$	$0.34885 \cdot 10^{-1}$
AWCYL-92-01	$-0.25832 \cdot 10^{-2}$	$0.38190 \cdot 10^{-1}$
AWCYL-92-02	$-0.11554 \cdot 10^{-1}$	0.13259
AWCYL-92-03	$-0.89050 \cdot 10^{-3}$	$0.58660 \cdot 10^{-1}$
E(. . . .)	$-0.33419 \cdot 10^{-2}$	$0.49771 \cdot 10^{-1}$

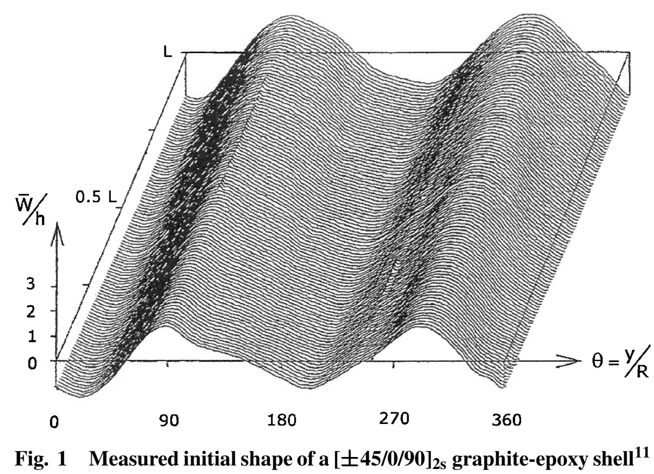

Fig. 1 Measured initial shape of a [±45/0/90]$_{2s}$ graphite-epoxy shell[11] (radius: 203.2 mm; length: 355.6 mm; wall thickness: 2.032 mm).

Table 2 Geometric and material properties of the NASA composite shells[11a]

Shell	Type	R, mm	t, mm	t_{ply}, mm	E_{11}, N/mm^2	E_{22}, N/mm^2	G_{12}, N/mm^2	ν_{12}
AWCYL-1-1	$[\pm 45/0/90]_s$	203.175	1.0140	0.1267	$1.2763 \cdot 10^5$	$1.1307 \cdot 10^4$	$6.0053 \cdot 10^3$	0.300
AWCYL-2-1	$[\pm 45/\mp 45]_{2s}$	203.429	2.0198	0.1262	$1.2809 \cdot 10^5$	$1.1307 \cdot 10^4$	$6.0260 \cdot 10^3$	0.300
AWCYL-3-1	$[\pm 45/0/90]_{2s}$	203.302	2.0117	0.1257	$1.2873 \cdot 10^5$	$1.1307 \cdot 10^4$	$6.0536 \cdot 10^3$	0.301
AWCYL-4-1	$[\pm 45/0 \times 4/\mp 45]_s$	203.048	1.9507	0.1219	$1.3279 \cdot 10^5$	$1.1307 \cdot 10^4$	$6.2466 \cdot 10^3$	0.299
AWCYL-5-1	$[\pm 45/90 \times 4/\mp 45]_s$	203.378	2.0157	0.1260	$1.2835 \cdot 10^5$	$1.1307 \cdot 10^4$	$6.0329 \cdot 10^3$	0.300
AWCYL-11-1	$[\pm 45/0/90]_{2s}$	203.200	2.0320	0.1270	$1.2755 \cdot 10^5$	$1.1307 \cdot 10^4$	$5\,9984 \cdot 10^3$	0.3
AWCYL-92-01	$[\pm 45/0 \times 2]_{2s}$	203.200	1.0160	0.1270	$1.2755 \cdot 10^5$	$1.1307 \cdot 10^4$	$5.9984 \cdot 10^3$	0.3
AWCYL-92-02	$[\pm 45/90 \times 2]_s$	203.200	1.0160	0.1270	$1.2755 \cdot 10^5$	$1.1307 \cdot 10^4$	$5\,9984 \cdot 10^3$	0.3
AWCYL-92-03	$[\pm 45/0/90]_s$	203.200	1.0160	0.1270	$1.2755 \cdot 10^5$	$1.1307 \cdot 10^4$	$5.9984 \cdot 10^3$	0.3

[a]For all shells, $L = 355.6$ mm.

For the buckling load calculations, shell AWCYL-11-1 was used. The imperfection model of Eq. (6) was specialized to the most critical long-wave imperfection combination with the axisymmetric mode $i = 2$ and the asymmetric mode $k = 1$ and $\ell = 6$. In particular, the strong coupling condition $i = 2k$ is enforced for this set of imperfection mode shapes. This condition for strong coupling between an asymmetric and an axisymmetric imperfection shape was first identified by Koiter and has been shown to cause a significant reduction in the buckling load of a compression-loaded cylinder. The corresponding eigenvalues are $\lambda_{ci} = 5.150$ and $\lambda_{c,k\ell} = 0.401$. In the probabilistic analysis, it is convenient to renormalize these eigenvalues so that the critical (the lowest) renormalized eigenvalue is equal to 1.0. Thus,

$$\rho_{ci} = \lambda_{ci}/0.398 = 12.932, \qquad \rho_{c,k\ell} = \lambda_{c,k\ell}/0.398 = 1.007$$

To apply the FOSM method, the mean buckling load has to be calculated first. Using the imperfection model of Eq. (6), with the mean values given in Eq. (12), yields $E(\Lambda_s) = 0.896$.

Thus,

$$E(Z) = E(\Lambda_s) - \rho = 0.896 - \rho \tag{14}$$

where ρ is the renormalized load parameter defined as

$$\rho = \lambda/\lambda_c^m \tag{15}$$

and $\lambda_c^m = 0.398$ is the critical (lowest) normalized buckling load of shell AWCYL-11-1 used for the numerical calculations. The variance Z is given by

$$\mathrm{var}(Z) = \mathrm{var}(\Lambda_s) \sim \sum_{j=1}^{2}\sum_{k=1}^{2}\left(\frac{\partial\Lambda_s}{\partial\xi_j}\right)\left(\frac{\partial\Lambda_s}{\partial\xi_k}\right)\mathrm{cov}(X_1, X_2) \tag{16}$$

The calculation of the derivatives $\delta\Lambda_s/\delta\xi_j$ is performed numerically by using the following numerical differential formulas evaluated, at values of $\xi_j = E(X_j)$:

$$\frac{\partial\Lambda_s}{\partial\xi_1} = \frac{\Lambda_s(\xi_1 + \Delta\xi_1, \xi_2) - \Delta_s(\xi_1, \xi_2)}{\Delta\xi_1} \tag{17}$$

$$\frac{\partial\Lambda_s}{\partial\xi_2} = \frac{\Lambda_s(\xi_1, \xi_2 + \Delta\xi_2) - \Lambda_s(\xi_1, \xi_2)}{\Delta\xi_2} \tag{18}$$

For the increment of the random variable, 1% of the original mean value of the corresponding equivalent Fourier coefficient is used, so that $\Delta\xi_j = 0.01E(X_j)$. The calculated derivatives are

$$\frac{\partial\Lambda_s}{\partial\xi_1} = 0.7481, \qquad \frac{\partial\Lambda_s}{\partial\xi_2} = -1.3622 \tag{19}$$

Next, using the sample variance-covariance matrix of Eq. (13), the variance of Z is calculated as

$$\mathrm{var}(Z) = 0.00246 \tag{20}$$

Hence, the reliability index β is

$$\beta = \frac{E(Z)}{\sqrt{\mathrm{var}(Z)}} = \frac{E(\Lambda_s) - \rho}{\sigma_Z} = \frac{0.896 - \rho}{0.0496} \tag{21}$$

Finally, the reliability is calculated directly from Eq. (4), that is,

$$R(\rho) = 1 - P_f(\rho) = 1 - F_Z(0) = \tfrac{1}{2} + \mathrm{erf}(\beta) = \phi(\beta) \tag{22}$$

where $\phi(\beta)$ is the standard normal probability distribution function. This result is plotted in Fig. 2. Notice that for a reliability of 0.99999, one obtains an improved, renormalized knockdown factor of $\rho_a = \lambda_a/\lambda_c^m = \lambda_a/0.398 = 0.68$. Results from the FOSM are compared to similar results obtained from a Monte Carlo method in Fig. 2. The Monte Carlo method is based on 5000 realizations or sample points as compared to the three analyses required to perform the FOSM method. These results indicate that the results obtained from the simplified FOSM method are conservative in the high-reliability region of the reliability curve, for example, $R(\rho)$ greater than 0.4.

A comparison of the improved, scientific knockdown factor ρ_a, the traditional empirical knockdown factor λ for isotropic shells, and the corresponding experimental buckling loads P_{exp} is presented in Table 3. As can be seen from the results, the reliability-based improved knockdown factor ρ_a in most cases yields a safe estimate of the load-carrying capacity of the shells tested if failure is caused by elastic instability. In addition, the results indicate that the new knockdown factors can be much less conservative than the corresponding traditional empirical knockdown factors for these shells based on the NASA design monograph for isotropic shells. For example, the traditional design approach for shell AWCYL-1-1 predicts a buckling

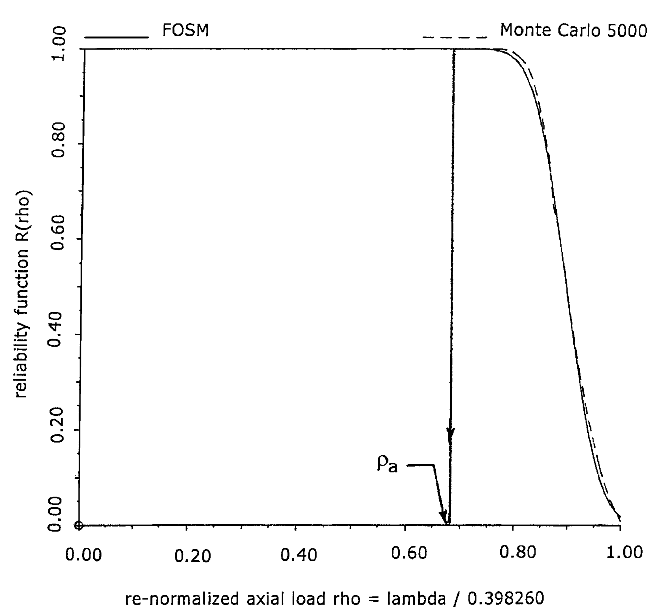

Fig. 2 Reliability of the group of nine composite shells[9] calculated using Koiter's modified two-mode imperfection model.

Table 3 Theoretical and experimental buckling loads of axially compressed composite shells[11]

Shell	N_{cl}, lb/mm	λ_c^m	λ	ρ_a	λP_c, N	$\rho_a P_c$, N	P_{exp}, N
AWCYL-1-1	−391.99	0.3660	0.471	0.68	−86,264	−124,546	−134,176
AWCYL-2-1	−1560.40	0.2225	0.580	0.68	−257,441	−301,825	−329,057
AWCYL-3-1	−1554.90	0.3980	0.579	0.68	−457,744	−537,594	−657,265
AWCYL-4-1	−1505.93	0.3186	0.575	0.68	−351,957	−416,229	−558,438
AWCYL-5-1	−1557.37	0.3397	0.580	0.68	−392,128	−459,737	−407,733[a]
AWCYL-11-1	−1568.68	0.3983	0.581	0.68	−463,424	−542,389	−676,574
AWCYL-92-1	−392.16	0.2659	0.471	0.68	−62,702	−90,526	−123,607
AWCYL-92-2	−392.16	0.2919	0.471	0.68	−68,841	−99,387	−142,005
AWCYL-92-3	−413.37	0.3561	0.471	0.68	−88,524	−127,802	−152,040

[a]Premature material failure.

load 36% lower than the experimentally measured buckling load. In contrast, the new probability-based knockdown factor predicts a buckling load that is only 7% less than the experimental buckling load.

The premature failure of specimen AWCYL-5-1 was caused by a material failure near a significant manufacturing defect in the form of a 0.15-in.-wide lamina ply gap. However, in such instances where there exists a significant defect as indicated here, one would expect quality assurance guidelines would reject such a part. In fact, such quality guidelines already exist in the MIL 17 Handbook on composite materials and restrict lamina ply gaps to be less than 0.1 in. wide. On the other hand, one might be able to account for such manufacturing defects in the design process by developing scientific-based knockdown factors for various types of manufacturing defects common to a specific manufacturing process. With this type of design knockdown factor information, a designer can make an informed decision on whether to invest time and money into improving the manufacturing process to minimize the defects or to settle for the knockdown associated with the defect.

Conclusions

A probability-based analysis method for predicting buckling loads of compression-loaded composite cylinders has been presented. It has been demonstrated that a database containing information about the specimen geometry and material properties and measured initial imperfections for a group of specimens can be used successfully to calculate improved, less conservative, buckling load knockdown factors, as compared to the lower-bound knockdown factors currently used in industry design practice. A first-order second-moment (FOSM) method and a Monte Carlo method were both used to evaluate the probability integral and a comparison of the results from both methods indicated that the FOSM method can be conservative for high values of reliability. Furthermore, the results show that the reliability-based knockdown factor ρ_a always yields a safe estimate of the buckling load of the cylinder specimens if failure is caused by elastic instability.

The probability-based analysis procedure presented in this study can be used to form the basis for a shell analysis and design approach that includes the effects of initial geometric imperfections on the buckling load of the shell. In particular, a probability-based analysis procedure can address some of the critical shell-buckling design criteria and design considerations for stability-critical shell structures without resorting to the traditional empirical shell design approach that generally leads to overly conservative designs.

It is anticipated that, for applications where the total weight of the structure is one of the critical parameters (e.g., aerospace structures), there will be a chance for definite improvement in the design process with the use of a new probabilistic design procedure. It is felt that the small added effort involved in systematically carrying out the required initial imperfection surveys will be fully justified by the overall cost savings and by producing improved, less conservative and more reliable shell structures. However, to develop a validated probabilistic design procedure for buckling-sensitive structures, there is a need for additional systematic combined experimental and analytical or numerical results. Only then can the benefits be reaped from the many years of concentrated shell buckling research of the late 1960s and early 1970s. It is the authors' opinion that the technology now exists for such an undertaking.

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